

$$T\{\phi_x \phi_1 \phi_2\} = \phi_x T\{\phi_1 \phi_2\} \theta(t-t_1) \theta(t-t_2) \quad A$$

$$+ \phi_1 T\{\phi_x \phi_2\} \theta(t_1-t) \theta(t_1-t_2) \quad B$$

$$+ \phi_2 T\{\phi_x \phi_1\} \theta(t_2-t) \theta(t_2-t_1) \quad C$$

$$\partial_t [\theta(t-t_1) \theta(t-t_2)] = \delta(t-t_1) \theta(t-t_2) + \theta(t-t_1) \delta(t-t_2)$$

$$\rightarrow \partial_t A = \phi_x T\{\phi_1 \phi_2\} \delta(t-t_1) \theta(t-t_2)$$

$$+ \phi_x T\{\phi_1 \phi_2\} [\delta(t-t_1) \theta(t-t_2) + \theta(t-t_1) \delta(t-t_2)]$$

$$\partial_t B = \phi_1 T\{\phi_x \phi_2\} \delta(t_1-t) \theta(t_1-t_2)$$

$$- \phi_1 T\{\phi_x \phi_2\} \delta(t_1-t) \theta(t_1-t_2)$$

$$\partial_t C = \phi_2 T\{\phi_x \phi_1\} \theta(t_2-t) \delta(t_2-t_1)$$

$$- \phi_2 T\{\phi_x \phi_1\} \delta(t_2-t) \theta(t_2-t_1)$$

$$= \delta(t-t_1) [\phi_x T\{\phi_1 \phi_2\} \theta(t-t_2) - \phi_1 T\{\phi_x \phi_2\} \theta(t_1-t_2)]$$

$$+ \delta(t-t_2) [\phi_x T\{\phi_1 \phi_2\} \theta(t-t_1) - \phi_2 T\{\phi_x \phi_1\} \theta(t_2-t_1)]$$

$$+ \cancel{\phi_x T\{\phi_1 \phi_2\}} + T\{\phi_x \phi_1 \phi_2\}$$

$$= T\{\phi_x \phi_1 \phi_2\}$$

Check of
Schwarz
Eq. 7.12

Vanish after some
quick examination

$$T\{\dot{\phi}_x \phi_1 \phi_2\}$$

$$= \dot{\phi}_x T\{\phi_1 \phi_2\} \theta(t-t_1) \theta(t-t_2) \quad \text{A}$$

$$\phi_1 T\{\dot{\phi}_x \phi_2\} \theta(t_1-t) \theta(t_1-t_2) \quad \text{B}$$

$$\phi_2 T\{\dot{\phi}_x \phi_1\} \theta(t_2-t) \theta(t_2-t_1) \quad \text{C}$$

$$\partial_t A = \ddot{\phi}_x T\{\phi_1 \phi_2\} \theta(t-t_1) \theta(t-t_2) \quad \checkmark$$

$$+ \dot{\phi}_x T\{\phi_1 \phi_2\} \left[\delta(t-t_1) \theta(t-t_2) + \theta(t-t_1) \delta(t-t_2) \right]$$

$$\partial_t B = \phi_1 T\{\ddot{\phi}_x \phi_2\} \theta(t_1-t) \theta(t_1-t_2) \quad \checkmark$$

$$-i\hbar \delta^d(x-x_2) \phi_1 \theta(t_1-t) \theta(t_1-t_2) \quad \checkmark$$

$$- \phi_1 T\{\dot{\phi}_x \phi_2\} \delta(t_1-t) \theta(t_1-t_2) \quad \checkmark$$

$$\partial_t C = \phi_2 T\{\ddot{\phi}_x \phi_1\} \theta(t_2-t) \theta(t_2-t_1) \quad \checkmark$$

$$-i\hbar \delta^d(x-x_1) \phi_2 \theta(t_2-t) \theta(t_2-t_1) \quad \checkmark$$

$$- \phi_2 T\{\dot{\phi}_x \phi_1\} \delta(t_2-t) \theta(t_2-t_1) \quad \checkmark$$

$$= \cancel{A} + \cancel{B} + \cancel{C} + T\{\ddot{\phi}_x \phi_1 \phi_2\} - i\hbar \delta^d(x-x_2) \phi_1 \theta(t_1-t) \theta(t_1-t_2) \\ - i\hbar \delta^d(x-x_1) \phi_2 \theta(t_2-t) \theta(t_2-t_1)$$

$$+ \delta(t-t_1) \left[\dot{\phi}_x T\{\phi_1 \phi_2\} \theta(t-t_2) - \phi_1 T\{\dot{\phi}_x \phi_2\} \theta(t_1-t_2) \right] \quad \leftarrow \begin{array}{l} \text{vanish} \\ \text{by inspection} \end{array}$$

$$+ \delta(t-t_2) \left[\dot{\phi}_x T\{\phi_1 \phi_2\} \theta(t-t_1) - \phi_2 T\{\dot{\phi}_x \phi_1\} \theta(t_2-t_1) \right] \quad \leftarrow$$

This confirms it.